Abstract: The history of induction motor started when Hans Christian Oersted discovered the magnetic effects of electric current in 1820. Induction motors are perhaps the most widely used electric motors in industry. They offer reasonable performance, a manageable torque-speed curve, stable operation under load and satisfactory efficiency. The aim of the paper is to investigate the performance characteristic of squirrel cage induction motor. Simulation and experimental results are presented and compared to validate the mathematical model of motor. The purpose of this study has been to devise a mathematical model, which can reliably predict the steady state performance of PWM inverter fed cage induction motor drive. The model has been developed on coupled circuit approach of the motor and in terms of measurable parameters of the system. The mathematical model developed is in form of differential equation and is used for implementation. The operating frequency has been selected as 50Hz. The steady state performance is computed under full load. The sources considered are sine wave supply and PWM supply. The characteristics of the motor under sinusoidal supply operation are obtained. For sinusoidal PWM inverter the steady state performance is checked. Performance of motor has been presented into a set of output graphs. The output graphs permit analysis of various motor parameters. MATLAB has been chosen for this paper because it is flexible and has various built in tools designed test, measurement and control.

Keywords: Mathematical model, PWM inverter, MATLAB/SIMULATION.

I. INTRODUCTION

The AC drive technology has fast torque response, speed accuracy and simple in design. AC drives are light weight, inexpensive and has low maintenance compared to DC drives. For variable speed applications, AC motors require power converters and AC voltage controllers in order to control frequency, voltage and current. The three phase induction motor is the workhorse of modern industry. Three-phase induction motors are commonly used in adjustable-speed drives. Computer-based Modeling and simulation of induction machine has opened new horizons for performance analysis. A good mathematical model can help in predicting the behaviour of an induction machine under different operating conditions and in selecting the appropriate machine for a specific operation.

II. PWM INVERTER

An inverter is a device that converts dc power into ac power. This can be broadly classified into two types: Voltage Source Inverter (VSI) and Current Source Inverter (CSI). For controlling the speed of an AC drive the flux should be kept constant, i.e. the air gap voltage to frequency (E/f) ratio should be kept constant. Since we vary the frequency to control the speed, hence voltage should be varied accordingly to keep E/f ratio constant. This voltage variation can be achieved in three ways:

1. By variation of the alternating voltage output from the inverter.
2. By variation of the direct voltage input to the inverter.
3. By switching techniques within the inverter circuit.

There exists a number of techniques for implementing pulse width modulation in power inverters, some of popular techniques are:

1. Single pulse modulation
2. Multiple pulse modulation
3. Square wave PWM
4. Sinusoidal PWM

III. MATHEMATICAL MODEL OF PWM INVERTER FED CAGE INDUCTION MOTOR DRIVE

This topic presents the generalized equations describing the behaviour of pulse width modulated fed squirrel cage induction motor drive based oil the coupled circuit approach. The system has been analysed in a synchronously rotating reference frame. With the help of d-q transformation of variables basic equations for induction motor, are developed in per unit system. The mathematical model so developed, has been used for analysing the drive under sinusoidal as well as PWM input.

3.1 INTRODUCTION

The PWM output voltage waveforms of the three-phase system are, generated based on sinusoidal pulse width modulation technique. These voltages are then transformed into d-q components and regarded as forcing functions in the d-q model of an induction motor. The model has been developed neglecting hysteresis and eddy current losses.

3.2 MATHEMATICAL MODEL OF INDUCTION MACHINE

The generalized equations describing the behaviour of an induction motor under steady state and transient conditions are established by considering it as an elementary two pole idealized machine.
Mathematical Model of Induction Motor in matrix form is expressed as

\[
\begin{bmatrix}
\psi_{as} \\
\psi_{bs} \\
\psi_{cs} \\
\psi_{ar} \\
\psi_{br} \\
\psi_{cr}
\end{bmatrix} =
\begin{bmatrix}
L_{as} & 0 & 0 & L_1 & L_2 & 0 \\
0 & L_{bs} & 0 & L_2 & L_1 & 0 \\
0 & 0 & L_{cs} & L_3 & L_2 & L_1 \\
L_4 & L_2 & L_3 & L_{rr} & 0 & 0 \\
L_5 & L_4 & L_2 & 0 & L_{rr} & 0 \\
L_2 & L_3 & L_1 & 0 & 0 & L_{rr}
\end{bmatrix}
\begin{bmatrix}
i_{as} \\
i_{bs} \\
i_{cs} \\
i_{ar} \\
i_{br} \\
i_{cr}
\end{bmatrix}
\]

A mathematical model has been developed from a three phase PWM inverter. The model developed can be used to predict the dominant feature of the drive under steady state and transient conditions.

IV. STEADY-STATE ANALYSIS

This section deals with steady state analysis of PWM signal fed, cage induction motor drive. Using mathematical model, developed, performance characteristics of the drive under steady state are, obtained for comparison, corresponding performance. Characteristics of the motor when fed from sinusoidal supply are also presented.

4.1 INTRODUCTION

The PWM voltage is considered as forcing function to the coupled circuit model of induction motor and waveforms of the motor currents are obtained in time domain. This requires the mathematical model of the drive to be solved through numerical techniques. From the initial standstill conditions the motor is allowed to build up under a given load torque until steady state is reached. The steady state is identified when the motor current waveform successively exhibits identical cycles. The voltage-current waveforms are then used to compute the steady-state performance in time-domain. This analysis is carried out at a selected frequency of 50 Hz and at no and full load condition.

4.2 PERFORMANCE OF THE TEST MOTOR ON SINUSOIDAL SUPPLY

In this section steady-state performance of the specific induction motor, is presented with, the sinusoidal supply to establish its behaviour. Performance curves are obtained under nominal supply (rated voltage and rated frequency) shown in fig 4.1. This curve is used to determine motor performance at a given frequency for comparison when the motor is fed from PWM inverter supply.

4.2.1 PERFORMANCE UNDER NOMINAL VOLTAGE AND FREQUENCY

The steady state performance curves have been obtained using equivalent circuit of the motor. The equivalent circuit parameters of the test motor, given in the Appendix, are obtained through light running and blocked rotor tests. The test motor's nameplate data are as shown in Table 4.2. Since the nameplate current refers to full load operation, it has been used for identifying the full load operating point on the performance curves. Fig 4.1 shows the computed steady state performance from equivalent circuit approach with respect to speed in the entire sub synchronous region, from standstill to synchronous speed, when the motor is operated from rated voltage and frequency supply. The full load operating points have also been identified on theses characteristics and are reproduced in Table 4.2. It may be noted that the calculated power output is a little more than the nameplate value. This is due to neglecting friction and windage losses and core losses in calculations.

<table>
<thead>
<tr>
<th>Connection</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Line Voltage</td>
<td>415.0 volts</td>
</tr>
<tr>
<td>Nominal Line Current</td>
<td>4.9 Amps</td>
</tr>
<tr>
<td>Nominal Power Output</td>
<td>2.2KW</td>
</tr>
<tr>
<td>Nominal Speed</td>
<td>1400 RPM</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Supply Voltage</th>
<th>In actual unit</th>
<th>In per unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>415.00 volts</td>
<td>0.707</td>
<td></td>
</tr>
<tr>
<td>50.00Hz</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>4.90 A</td>
<td>1.225</td>
<td></td>
</tr>
<tr>
<td>16.56Nw-m</td>
<td>0.740</td>
<td></td>
</tr>
<tr>
<td>2.41KW</td>
<td>0.686</td>
<td></td>
</tr>
<tr>
<td>2.81KW</td>
<td>0.800</td>
<td></td>
</tr>
<tr>
<td>0.686</td>
<td>0.860</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.800</td>
<td></td>
</tr>
<tr>
<td>1440 RPM</td>
<td>0.960</td>
<td></td>
</tr>
<tr>
<td>367.00volts</td>
<td>0.626</td>
<td></td>
</tr>
</tbody>
</table>

From Fig 4.1 (a) it is noted that motor's starting torque is 1.04 p.u. which is 1.4 times the full load torque. The full load slip
is 0.072 p.u. The starting current in Fig 4.1 (g) is 4.8 p.u. which is 3.92 times the full load current. Figs. 4.1 (g) show variation of power input and power output respectively with respect to speed. Curves 4.1 (c and e) show variations of power input and power output figures (b and d) shows power factor and efficiency, the full load values of them being 0.796 pu. and 0.856 p.u. Fig 4.1 (f) shows air gap voltage. The air gap voltage is seen to be 0.664 p.u. (or 389.7 volts) under no load which slightly drops to 0.626 p.u. (367 volts) under full load condition.

4.3 OPERATION ON PWM INVERTER SUPPLY

The output voltage of PWM inverter is non-sinusoidal. The fundamental component of this voltage can be utilized to select the value of Supply voltage.

4.3.1 VOLTAGE TO FREQUENCY CONTROL IN PWM INVERTER

In present work both voltage and frequency at the output of inverter are varied to achieve speed control under v/f control to maintain high torque capability at all frequencies. The ratio v/f is chosen corresponding to the rated voltage and frequency. In PWM inverters, amplitude of fundamental output voltage is directly proportional to the modulation index 'm', which is defined as:

\[ M = \frac{V_r}{V_c} = \text{Amplitude of modulating wave/amplitude of carrier wave}. \]

The frequency ratio or carrier ratio is defined as:

\[ K = \frac{f_c}{f} = \text{Carrier frequency / Frequency of modulating wave}. \]

For the frequency ratio 'K' is chosen to be multiple of three identical phase voltage waveforms at the inverter output are produced, resulting in a balanced three phase system.

In sinusoidal PWM, three phase sinusoidal voltages are compared with a carrier wave to obtain PWM pattern. This is illustrated in Fig 4.2 (a) compares sinusoidal modulating wave of phase 'a' with a triangular carrier waveform symmetrically placed about zero reference axis. In this illustration the carrier ratio is chosen to be 20 and the corresponding modulating index selected is 1.0. This comparison is used to generate a pole voltage (pva) waveform as shown in Fig 4.2 (b) which swings between \(+ (V_d / 2)\) to \(- (V_d / 2)\), where \(V_d\) is the dc link voltage at the input of inverter. In present case the value of \(V_d\) 1.5 P.u. The logic employed in generating the pole voltages is that pole voltage is \(+ (V_d / 2)\) at those instants when amplitude of modulating wave is greater than that of the carrier wave, otherwise it is \(- (V_d / 2)\). Thus the pole voltage has only two levels (either positive or negative) and is called two-level PWM waveform. The pole voltage waveform is considered as the voltage of a phase with respect to midpoint of the dc source \(V_d\). A 3-level PWM pole voltage waveform is also commonly employed.

Fig 4.2 (c) shows the comparison of sinusoidal modulating waveform. Of phase 'B' with carrier wave. The modulating waveforms of phase B is 120° lagging in time phase with respect to modulating wave of phase 'A'. This comparison yields pole voltage waveform. 'pvb' as shown in the Fig. 4.2 (d). The line voltage \(V_{ab}\) is obtained as \(V_{ab} = pva - pvb\).

The waveform of other line voltages \(V_{bc}\) and \(V_{ca}\) at the output of inverter are obtained in a similar manner. Thus,

\[ \begin{align*}
V_{bc} &= pvb - pvc \\
V_{ca} &= pvc - pva
\end{align*} \]

For wye-connected load at the output of inverter the phase voltages may also be from pole voltages as

\[ \begin{align*}
V_a &= (2 pva - pvb - pvc)/3 \\
V_b &= (2 pvb - pvc - pva)/3 \\
V_c &= (2 pvc - pva - pvb)/3
\end{align*} \]

The phase voltages of phase 'A' 'Vas' and the line voltage \(V_{ab}\) obtained in this manner are shown in fig 4.2 (e) and (f) respectively.

The amplitude of fundamental line to neutral voltages for a star connected load fed from a PWM inverter is found to be.

\[ V_l = m \times \frac{V_d}{2} \]

This equation points to two ways of achieving, control of output fundamental voltage \(V_l\) (i) Keeping the modulating index \(m = 1.0\) and varying \((V_d)\) supply voltage at the inverter input which results in variation of output fundamental voltage \(V_l\) and (ii) for a given supply voltage, varying modulation index 'm' between 1.0 and 0.0. In the present work, performance of the drive under modulation index m 1.0 is observed.

If \(V_d\) is chosen 1.5 p.u. as in Fig 4.2, equation (4.1) becomes \(V_l = 0.75\).

4.4 SYSTEM EQUATIONS UNDER STEADY STATE

In case of sinusoidal input to the motor, motor voltages and currents attain steady ac values under steady state. When referred to synchronously rotating d-q reference frame they appear to be dc quantities and their time derivative become zero. However in case of PWM inverter fed induction motor drive the input voltage is non-sinusoidal and therefore for steady state. The dynamic equations of the motor that are nonlinear may be solved by numerical analysis method to get steady state currents.

4.4.1 DIGITAL SIMULATION

The dynamic state of the induction motor can be represented by the voltage-current relations in the motor and may be expressed in the following form

\[ [V] = [R] [I] + 1/\omega [X] [P] \]
Or

\[ P[i] = \omega [X]^{-1} [R] [i] + [X]^T [V] \]

Where \([V]\) is the voltage vector

\([i]\) is the current vector

\([R]\) is the impedance matrix free of \(p\) terms

\[
[R] = \begin{bmatrix}
R_{ss} & -\omega L_{ss} & 0 & -\omega L_{12} \\
\omega L_{ss} & R_{ss} & \omega L_{12} & 0 \\
0 & -S\omega L_{12} & R_{rr} & -S\omega L_{rr} \\
S\omega L_{12} & 0 & -S\omega L_{rr} & R_{rr}
\end{bmatrix}
\]

Rewriting equation 3.44

\[
P(\omega r) = 0.5 \frac{(T_e - T_L)}{H}
\]

Equations (4.5) and (4.6) constitute a set of five non-linear differential equations. This set can be efficiently solved on a digital computer using numerical integration technique.

Fourth order Runge-Kutta method of numerical integration is adopted here. The accuracy of integration depends on the integration interval; smaller the interval, greater is the accuracy. A step interval of 0.00002 second is selected in the present work.

4.4.2 COMPUTATION PROCESS IN PWM INVERTER

The computation process adopted in present work has been illustrated through flow charts shown in present Fig 4.3 to 4.5. It begins by taking initial values of rotor speed and machine phase currents and hence \(i_d\), \(i_q\), \(i_dr\), \(i_qr\) as zero. The value of step length, base, frequency, operating frequency, initial values of applied load torque and modulation index \(m\) are also provided as input. The phase voltages \(V_a\), \(V_b\), and \(V_c\) are calculated using 'PWM' given in flow chart of Fig 4.4. It generates three phase sine voltages and carrier wave of given carrier ratio. The PWM waveform is generated by comparing the amplitudes of three sinusoidal voltages and the carrier wave amplitudes using the concept of two level modulation. The test motor is delta connected, its line, and phase voltages are to be obtained from the converter output. The line voltages \(V_a\), \(V_b\) \(V_c\) and phase voltages \(V_a\), \(V_b\), \(V_c\) are calculated from the inverter pole voltages. Using d-q transformation, the line voltages are then transformed into \(V_{ds}\) and \(V_{qs}\). Equation (4.4) are then used in the main program of Fig 4.2 to calculate the currents \(i_{ds}\), \(i_{qs}\), \(i_{dr}\), and \(i_{qr}\). The d-q transformation is used once again to obtain phase currents \(i_a\), \(i_b\), and \(i_c\) as shown in the Fig 4.5. The next cycle of calculation starts with the above-calculated values as new initial values of variables, and 'PWM' calculation is done once again. In this manner, using the chosen step length, the voltages and currents are computed until the machine reaches steady state. At the end of each cycle, the torque developed by the motor \(T_e\) and its speed \((\omega r)\) is also computed for given Load torque \((TL)\).

4.4.3 VOLTAGE AND CURRENT EXPRESSIONS FOR MOTOR

The phase currents are related to d-q current components in accordance with transformation defined in equations (3.21) to (3.24). Rewriting these relations for stator,

\[
ids = \frac{2}{3} [i_{as}\cos \theta + i_{bs}\cos(\theta - 2\pi/3) + i_{cs}\cos(\theta + 2\pi/3)]
\]

\[
iqs = -\frac{2}{3} [i_{as}\sin \theta + i_{bs}\sin(\theta - 2\pi/3) + i_{cs}\sin(\theta + 2\pi/3)]
\]

Expression (4.8) and (4.9) can be written in inverse manners as

\[
i_{as} = i_ds\cos \theta - i_{qs}\sin \theta
\]

\[
i_{bs} = i_ds\cos(\theta - 2\pi/3) - i_{qs}\sin(\theta - 2\pi/3)
\]

\[
i_{cs} = i_ds\cos(\theta + 2\pi/3) - i_{qs}\sin(\theta + 2\pi/3)
\]

4.7
Since the stator windings are delta connected, the line currents and phase currents are not the same.

\[ \begin{align*} 
\text{ial} &= \sqrt{3} \, i_a \\
\text{i b1} &= \sqrt{3} \, i_{b1} \\
\text{i c1} &= \sqrt{3} \, i_{c1}
\end{align*} \]

### 4.4.4 Expressions for Input Power, Output Power and Efficiency

Power output can be determined from the electromagnetic torque \( T_e \) and angular shaft speed at every step by

\[ P_{\text{out}} = T_e \times \omega \times r \, (\text{mech}) \]

\[ p.f = \text{Pin} / s \]

\[ \eta = \text{Pout} / \text{Pin} \]

## V. Simulation Results and Discussion

Performance of the drive has been obtained with rated frequency using the computational results outlined in the previous section. The results of full load operation of the motor are presented here for operation on

(a) Sinusoidal supply

(b) PWM inverter supply

The waveforms of motor voltage, current, torque, input power and output power are obtained in time domain and are presented for operation in time domain, and are presented for operation at typical frequency of 50 Hz. Full load operation is considered by choosing a load torque \( T_L = 0.74 \, \text{p.u.} \), inertia constant \( H \) is taken as 0.1024 sec. The computation is also done considering saturation of the magnetic circuit. At the given frequency, the selection of voltage to be applied to the motor is done as below.

(a) Sinusoidal supply

For \( v/f \) control, the voltage to be applied for an operating frequency of \( f1 \, \text{Hz} \) is found in the following manner.

Chosen \( v/f \) ratio = \( V_{L-L} \, \text{(RMS)} / f = 400 \, \text{V} / 50 \, \text{Hz} = 8 \)

At frequency \( f1 \, \text{Hz} \)

\[ V_{L-L} \, \text{(rms)} = 8 \times f1 / \sqrt{3} \]

Or \( V_{\text{ph}} \, \text{(rms)} = V_{L-L} \, \text{(rms)} / \sqrt{3} = 8 \times f1 / \sqrt{3} \)

Or \( V_{\text{ph}} \, \text{(peak)} = (\sqrt{2} / \sqrt{3}) \times 8 \times f1 / (\sqrt{2} / \sqrt{3}) \times 400 \)

Or \( V_{\text{ph}} \, \text{(peak)} = f1 / 50 \)

Thus, \( V_{\text{ph}} \, \text{(peak)} = 41.92 \) for \( f \, 50 \, \text{Hz} \)
(b) PWM inverter supply
For a dc link supply voltage Vd chosen equal to 1.5 P.u., per unit fundamental phase voltage (peak value) is obtained from equation (3.1) as

\[ V_1 = 0.75 \text{ m} \]

The modulation index is selected as \( m = 0.8 \) for 50 Hz
Two level modulation is adopted with a carrier ratio of 20
The simulation results for \( f = 50 \text{ Hz} \) operation, when the drive fed from sinusoidal supply or PWM inverter supply are given in the Figs. 4.6 and 4.7 respectively. Fig. 4.6 (a) shows the applied phase / line voltage to the delta connected stator of the motor. As already mentioned the peak value of phase/line voltage is has been set to base voltage 1.0 p.u. Fig. 4.6 (a and b) show electromagnetic torque developed and speed. Their steady state values being 0.74 p.u. and 1.0 p.u. respectively. Figs 4.6 (c and d) show phase and line currents, they are steady at 1.0 p.u and 1.74 p.u. Figs 4.6 (e and f) show input and output power, they are steady at 0.741 and 0.68 p.u.

Figs 4.7 (a - f) present the waveforms under PWM operation. For two level modulation, the peak value of pole voltage, line voltage/phase voltage are respectively Vd / 2 and Vd. In present case Vd is Chosen as 1.5 p.u. and therefore peak value of line voltage is 1.5 p.u.

Figs.4.7 (a and b) show electromagnetic torque and speed. Figs 4.7 (c and d) show phase and line current It is noticed that although the distortion is more in voltage waveform, the current waveform has less amount of distortion due to the fact the motor winding has high reactance to the applied harmonic voltage.

Fig 4.7 (e) shows the variation of input power to the motor. It reveals that the effect of harmonics on motor voltages and currents has an effect of input power too. The input power is obtained as the product of instantaneous voltages and currents. Its average value is almost equal to that of sinusoidal supply.

In the Fig.4.7 (f), the output power is obtained as the product of instantaneous torque and shaft speed. Since the oscillation in speed is very small, the output power follows the nature of torque. As compared, with the input power, oscillations in the output power have been reduced. Moreover the average power seems to be very close to that of sinusoidal supply. Resonant peak is reduced from 3.4 p.u. to 2.8 p.u. for torque while is, almost same for other parameters, settling time is reduced from 0.42 sec to 0.18 sec. Steady state performance is almost same for both the cases.

CONCLUSIONS
A mathematical model of Cage Induction motor drive has been developed in terms of measurable parameters of the system. Steady state performance of the motor is evaluated for sinusoidal supply and PWM inverter supply at nominal frequency. It is found that the performance of the motor is comparable in both the cases of supply and distortions are reduced as modulation index decreases in the PWM inverter line voltage. As carrier ratio is increased line current is nearly sinusoidal. Settling time is reduced with PWM inverter supply. The system is simulated on Digital computer, based on coupled circuit approach. A computer program using "MATLAB" software is developed here to solve the state equations using Fourth Order Runge - Kutta method to Simulate the performance of the drive.

The paper shows that MATLAB software package is highly suitable for the digital simulation of induction machine model

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APPENDIX

A- 1 Specifications of the test motor and base quantities
(I) Specifications of the test motor:
3 ph, 2.2 kW, 4 pole, 415 V, 50 Hz, 4.9 A, 1400 rpm
Delta connected cage induction motor
Combined inertia of the motor and the loading machine
J = 0.06 Kg-m2
U.

(II) Base values for various quantities:
Unit voltage, Vbase = peak of phase rated voltage
= 586.9 volts
Unit current, Ibase = peak of phase rated current
= 4.0 amp
Unit impedance, Zbase =Unit voltage/Unit current
= 146.7 ohms.
Unit power, Pbase = rated apparent power
= 3/2 *586.9 * 4.0 = 3.5 KW
Unit electrical angular speed = \( \Omega f = 100 \text{ rad./sec} \)
Unit mechanical speed = \( \Omega f = 50 \text{ rad./sec} \)
Unit torque, Te = unit power / unit mechanical speed
= 22.4 N-m

A- 2 Equivalent circuit parameters of the test motor
(At 50 Hz, sinusoidal supply)

Machine parameters:
Rs = Stator resistance per phase = 0.061 p. u.
Rr = Rotor resistance per phase = 0.075 p. u.
Xls = Stator leakage reactance per phase = 0.111 p. u.
X1r = Rotor leakage reactance per phase = 0.111 p. u.
X12 = Magnetizing reactance for starting unsaturated value of X12 is considered, Otherwise it is taken from saturation curve.